## HEAT TRANSFER IN A FLUIDIZED BED WITH ALLOWANCE FOR THE FILTRATION COMPONENT

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A two-phase model of surface-to-pseudoturbulent dispersed bed heat transfer is applied for calculating temperature fields in a fluidized bed. Calculation results are compared with experimental data.

The authors of [1] proposed a two-phase model of heat exchange between a surface and a fluidized bed. As before, we shall consider a fluidized bed to be any moving layer of disperse material which undergoes velocity and temperature pulsations [2]. It would be interesting to use the representations developed in [1] to study the temperature profiles in such a bed in the presence of gas filtration.

As an example, we will examine the problem of heat exchange between a flat surface and the fluidized bed in which it is submerged. Based on [1], the heat-transfer process can be represented as follows. Heat from the heating surface is transmitted by conduction to the gas which flows over it and thence (again by conduction) to the solid particles of the bed. Due to the pseudoturbulent pulsations, the particles diffuse from the surface of the bed to its core, carrying with them heat they have accumulated. The problem is examined in a one-dimensional approximation: The temperatures of the gas and particles depend only on the distance to the surface along the horizontal. In each vertical section, the gas entering the bed from below is heated to  $t_g$ , creating a convective flow  $vc\rho(t_g - t_o)/h = B(t_g - t_o)$ .

Then the mathematical problem is formulated as follows

$$\lambda_{\mathbf{g},\mathbf{e}} \frac{d^2 t_{\mathbf{g}}}{dx^2} - \alpha^* S\left(t_{\mathbf{g}} - t_{\mathbf{p}}\right) - B\left(t_{\mathbf{g}} - t_0\right) = 0, \tag{1}$$

$$\lambda_t \frac{d^2 t_p}{dx^2} + \alpha^* S(t_g - t_p) = 0.$$
<sup>(2)</sup>

System (1)-(2) will be solved with the following boundary conditions:

$$-\lambda_{g,e} \frac{dt_g}{dx}\Big|_{x=0} = q_0; \quad \frac{dt_p}{dx}\Big|_{x=0} = 0,$$
(3)

$$\frac{dt_{\mathbf{g}}}{dx}\Big|_{x=l} = \frac{dt_{\mathbf{p}}}{dx}\Big|_{x=l} = 0.$$
(4)

We will write Eqs. (1)-(4) in dimensionless form:

$$\frac{d^2\Theta_{\mathbf{g}}}{dY^2} - A_{\mathbf{t}}^2 \left(\Theta_{\mathbf{g}} - \Theta_{\mathbf{p}}\right) - \frac{Bd^2}{\lambda_{\mathbf{g},\mathbf{e}}} \left(\Theta_{\mathbf{g}} - 1\right) = 0, \qquad (5)$$

$$\frac{d^2\Theta_{\mathbf{p}}}{dY^2} + A_t^2 \frac{\lambda_{\mathbf{g},\mathbf{e}}}{\lambda_t} (\Theta_{\mathbf{g}} - \Theta_{\mathbf{p}}) = 0,$$
(6)

$$\frac{d\Theta_{\mathbf{g}}}{dY}\Big|_{Y=0} = -\frac{q_0 d}{\lambda_{\mathbf{g},\mathbf{g}}t_0}; \quad \frac{d\Theta_{\mathbf{p}}}{dY}\Big|_{Y=0} = 0, \tag{7}$$

$$\frac{-d\Theta_{\mathbf{g}}}{dY}\Big|_{Y=L} = \frac{-d\Theta_{\mathbf{p}}}{dY}\Big|_{Y=L} = 0.$$
(8)

To solve system (5)-(6), we will express  $\theta_g$  from (6) through  $\theta_p$ :

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$$\Theta_{\mathbf{g}} = -\frac{\lambda_{\mathbf{t}}}{A_{\mathbf{t}}^2 \lambda_{\mathbf{g},\mathbf{e}}} \Theta_{\mathbf{p}}^{''} + \Theta_{\mathbf{p}}.$$
(9)

Substituting (9) into (5), it is not difficult to obtain an equation for  $\theta_p$ :

$$\Theta_{\mathbf{p}}^{\mathrm{IV}} - a\Theta_{\mathbf{p}}^{''} + b\left(\Theta_{\mathbf{p}} - 1\right) = 0, \tag{10}$$

in which  $a = A_t^2 + A_t^2 \frac{\lambda_{g,e}}{\lambda_t} + B \frac{d^2}{\lambda_{g,e}}; \ b = A_t^2 B \frac{d^2}{\lambda_t}.$ 

Equation (10) can be solved by the method of characteristics [3]:

$$\Theta_{\mathbf{p}} = 1 + c_1 \exp(\lambda_1 Y) + c_2 \exp(\lambda_2 Y) + c_3 \exp(\lambda_3 Y) + c_4 \exp(\lambda_4 Y), \tag{11}$$
  
where  $\lambda_1 = \sqrt{\frac{a}{2} + \sqrt{\frac{a^2}{4} - b}}; \quad \lambda_3 = \sqrt{\frac{a}{2} - \sqrt{\frac{a^2}{4} - b}}; \quad \lambda_2 = -\lambda_1; \quad \lambda_4 = -\lambda_3.$ 

We obtain the expression for  $\boldsymbol{\Theta}_{\!\!\boldsymbol{g}}$  by substituting (11) into (9):

$$\Theta_{\mathbf{g}} = 1 + c_1 \left( 1 - \frac{\lambda_t \lambda_1^2}{\lambda_{\mathbf{g},\mathbf{e}} A_t^2} \right) \exp\left(\lambda_1 Y\right) + c_2 \left( 1 - \frac{\lambda_t \lambda_2^2}{\lambda_{\mathbf{g},\mathbf{e}} A_t^2} \right) \exp\left(\lambda_2 Y\right) + c_3 \left( 1 - \frac{\lambda_t \lambda_3^2}{\lambda_{\mathbf{g},\mathbf{e}} A_t^2} \right) \exp\left(\lambda_3 Y\right) + c_4 \left( 1 - \frac{\lambda_t \lambda_4^2}{\lambda_{\mathbf{g},\mathbf{e}} A_t^2} \right) \exp\left(\lambda_4 Y\right)$$

The constants  $c_1$ ,  $c_2$ ,  $c_3$ , and  $c_4$  in (11) and (12) are found from conditions (7) and (8). Simple transformations yield:

$$c_1 = -\frac{D \exp\left(-2\lambda_1 L\right)}{\lambda_1 \left[1 - \exp\left(-2\lambda_1 L\right)\right]};$$
(13)

$$c_2 = -\frac{D}{\lambda_1 \left[1 - \exp\left(-2\lambda_1 L\right)\right]}; \tag{14}$$

$$c_3 = \frac{D \exp\left(-2\lambda_3 L\right)}{\lambda_3 \left[1 - \exp\left(-2\lambda_3 L\right)\right]};$$
(15)

$$c_4 = \frac{D}{\lambda_3 \left[1 - \exp\left(-2\lambda_3 L\right)\right]}.$$
(16)

Substituting (13)-(16) into (11) and (12) gives us

$$\Theta_{\mathbf{p}} = 1 - \frac{D}{\lambda_1} \left\{ \frac{\exp\left(-\lambda_1 Y\right) + \exp\left[-\lambda_1 (2L - Y)\right]}{1 - \exp\left(-2\lambda_1 L\right)} \right\} + \frac{D}{\lambda_3} \left\{ \frac{\exp\left(-\lambda_3 Y\right) + \exp\left[-\lambda_3 (2L - Y)\right]}{1 - \exp\left(-2\lambda_3 L\right)} \right\};$$
(17)

$$\Theta_{g} = \Theta_{p} - \gamma D \left\{ \frac{\lambda_{3} \exp\left(-\lambda_{3} Y\right) \left[1 + \exp\left[-2\lambda_{3} \left(L - Y\right)\right]\right]}{1 - \exp\left(-2\lambda_{3} L\right)} - \frac{\lambda_{1} \exp\left(-\lambda_{1} Y\right) \left[1 + \exp\left[-2\lambda_{1} \left(L - Y\right)\right]\right]}{1 - \exp\left(-2\lambda_{1} L\right)} \right\}, \quad (18)$$

where

$$D = \frac{q_0 dA_{\mathbf{t}}^2}{\lambda_{\mathbf{t}} t_0 \left(\lambda_1^2 - \lambda_3^2\right)}; \quad \gamma = \frac{\lambda_{\mathbf{t}}}{A_{\mathbf{t}}^2 \lambda_{\mathbf{g}\cdot\mathbf{e}}}.$$

Calculation of the temperature fields in the fluidized bed from Eqs. (17) and (18) requires knowledge of  $A_t$  and  $\lambda_t$ .

The parameter  $A_t$ , which characterizes the rate of heat transfer across the phase boundary, was obtained earlier in [4]. It was shown that, for a packed bed of disperse material with a porosity  $\epsilon = 0.4$ ,  $A_t = \sqrt{6(1-\epsilon_f)Nu^*}=2$ . Since the porosity  $\epsilon_f$  of a fluidized bed is greater than the porosity of a packed bed, then  $A_t$  is less for a fluidized bed than for a packed bed.

The value of  $A_t$  was calculated for the above experiment, here assuming that Nu\* for a packed bed differs little from Nu\* for a fluidized bed. The value of  $\epsilon_f$  for the above fluidizing regime was determined in accordance with the formulas in [5].

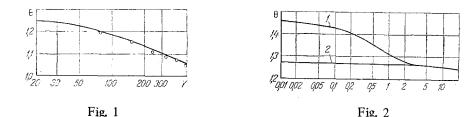


Fig. 1. Temperature profile along the horizontal in a fluidized bed according to (17) and test data: air filtration velocity  $v_r = 0.0835$  m/sec,  $\epsilon_f = 0.65$ ,  $A_t^2 = 2.351$ ,  $\lambda_t = 33$  W/m·K.

Fig. 2. Theoretical temperature profiles in the wall region: 1) gas temperature from Eq. (17); 2) particle temperature from Eq. (18).

The value of  $\lambda_t$  was determined on the basis of measurements of the temperature field in a fluidized bed. A 0.1-0.16-mm fraction of sand with an initial height  $h_0 = 0.01$  m was fluidized with air blown through a grate 0.31 × 0.065. A flat electric heater was placed in the middle of the bed, dividing it into two symmetrical parts. The temperature of the bed in the horizontal direction was measured with wire resistance thermometers at a certain distance from the heater surface, where  $t_g = t_p = t_h$ .

We can write the following for this section, without consideration of the features of heat transfer near the heater surface

$$\lambda_{t} \frac{d^{2} (t_{b} - t_{0})}{dx^{2}} - vc\rho (t_{b} - t_{0})/h = 0.$$
(19)

Solving this equation with the following boundary conditions:

$$\frac{d(t_{\mathbf{b}}-t_{\mathbf{0}})}{dx}\Big|_{x=0} = \frac{d\Theta_{\mathbf{b}}}{dx}\Big|_{x=0} = -\frac{q_{\mathbf{0}}}{\lambda_{\mathbf{t}}}, \quad \Theta_{\mathbf{c}}\Big|_{x=1} = \Theta_{\mathbf{k}},$$

we can obtain an expression for the bed temperature

$$\Theta_{\mathbf{b}} = \left\{ \left[ \Theta_{\mathbf{k}} - \frac{q_0}{\lambda_{\mathbf{t}} M} \exp\left(-Ml\right) \right] \exp\left(Mx\right) + \left[ \Theta_{\mathbf{k}} + \frac{q_0}{\lambda_{\mathbf{t}} M} \exp\left(Ml\right) \right] \exp\left(-Mx\right) \left[ \exp\left(Ml\right) + \exp\left(-Ml\right) \right]^{-1},$$
(20)

where  $M = \sqrt{B/\lambda_t}$ ;  $B = vc\rho/h$ .

The value of the effective horizontal thermal conductivity  $\lambda_t$  was found by comparing the test data with the solution of (20). The resulting value of  $\lambda_t$  was used to calculate temperature with Eqs. (17) and (18). To find the heat flux  $q_0$  and filtration sink B $\Theta_b$ , it is necessary to know the bed height. This was determined from the formulas in [5]. The quantity  $\lambda_{g,e}$  in (17)-(18), which is dependent on the thermal conductivity of the solid particles and the gas phase, was calculated from [6].

Figures 1 and 2 show experimental temperature profiles along the bed, as well as theoretical profiles obtained from (17) and (18). The good agreement between the profiles is evident from the figures.

Thus, the proposed model can be used to study heat transfer in fluidized beds in the presence of filtration. The model also makes it possible to evaluate the temperature field close to the surface of the heat exchanger (see Fig. 2), where experimental measurements are difficult.

It must be noted that the quantity  $\lambda_t$ , calculated from the experimental temperature profile, depends substantially on the proper selection of a height for the bed conducting the heat flow. Solution of this problem requires a separate investigation.

#### NOTATION

 $\lambda_t$ , effective thermal conductivity; t, temperature; x, coordinate along the bed; l, length of bed, m;  $\alpha^*$ , interphase heat-transfer coefficient;  $q_0$ , heat flux;  $\Theta = t/t_0$ , dimensionless temperature; Y = x/a, dimensionless coordinate; d, particle diameter, m; L = l/d, dimensionless length of bed;  $\epsilon_f$ , porosity; h, bed height, m; v, c,  $\rho$ , velocity, heat capacity, and density of fluidizing air; Nu\*, dimensionless interphase heat-transfer coefficient;  $\Theta_b = t_b - t_0$ , excess temperature of bed;  $t_0$ , temperature of fluidizing gas at bed inlet. Indices: g, gas; p, particles; t, turbulent; b, bed.

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# HEAT TRANSFER OF A SWIRLED FLOW OF GAS SUSPENSION IN A SHORT CHANNEL

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On the basis of generalized experimental data obtained by the method of local modeling, the article suggests a method of calculating the heat transfer of a swirled flow of gas suspension in a short channel.

Sharafutdinov et al. [1] examined the results of the experimental investigation of the heat transfer of a swirled flow containing dust particles of aluminum oxide 1-11  $\mu$ m in size in a short partitioned pipe, the partitioning corresponding to the chamber of a rocket engine. The heat-transfer coefficients were determined by the gradient method with the aid of a cylindrical experimental section 0.18 m long and with inner diameter d = 0.106 m. Swirling of the flow was effected by swirlers with straight vanes [2] which were mounted ahead of the cylindrical experimental section 2 at a distance of 0.135 m (Fig. 1). The length of the passive heat-insulated section was  $L_0 = 2.91$  m, the distance between the beginning of the heat-insulated pipe and the swirler was  $L_1 = 2.775$  m. Behind the cylindrical experimental section there was the adapter 3, 0.114 m long, and the nozzle section 4 whose profile consisted of circular arcs. The diameter of the critical nozzle section was  $d_{cr} = 25$  mm.

The experimental data of [1] were generalized by the method of local modeling in the form of the dependence of the correction  $\Psi_{s\varphi}$  to the standard law of heat exchange on the similarity parameter  $K_{pf}$  [3] expressing the influence of the distorting effect of the particles in the near-wall zone before their collision with the wall (influence of the primary effect of the particles) on the heat transfer. With the aid of the similarity parameter  $K_{pf}$  we were able to generalize the experimental data only for swirlers with angle of mounting the vanes  $\varphi_k = 15$ , 25, and 35°. For swirlers with  $\varphi_k = 45$  and 60° it was impossible to generalize the experimental data on the basis of the similarity parameter  $K_{pf}$  because with large angles the secondary effect of the particles (after their collision with the wall) on the gas phase has a considerable influence on the heat exchange, and the similarity parameter  $K_{pf}$  does not take this influence into account.

The present article submits the results of generalizing the experimental data of [1] taking into account the influence of the secondary distorting effect of particles in the near-wall zone on the heat exchange, and a method is suggested for calculating the heat transfer of a swirled two-phase stream in a short channel.

In the subsequent analysis we will proceed from the following assumptions.

1. The process of heat transfer of two-phase swirled stream is the sum of the process of heat transfer of the carrier gas phase distorted by particles, and the process of heat exchange of the particles with the wall upon direct contact.

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